Blowing Cables in Ducts Fixed at Regular Intervals

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Abstract
Installation of optical cables in ducts by blowing (with air) or by floating (with water) are widely used and well known techniques. However, some mysteries remain, e.g. what is the effect on the installation length when a (HDPE) duct is hanging, fixed with regular intervals (spans), and how does this depend on the span? In this paper an analysis is made. It is found that for constant temperature the blowing length is hardly affected for a span of 2 m or less (for 32/26 mm duct, span scaling proportionally to duct diameter) and when the duct is hung under tension this is also true for larger spans. But, when the duct is in an environment with varying temperatures (e.g. under bridges) a span of 1 m can already be critical (blowing length drops by ~1/3rd). The initial tension only helps a bit here. Creep of the duct and cable sag also have an effect and are treated qualitatively. Tests are proposed to check all details. Floating the cable helps to reach longer installation lengths, also because it may be used to cool the duct.

Keywords: Cable; optical; duct; installation; blowing; floating; fixed; clamped; regular intervals; span; bridges; temperature; cooling; tunnels; sea; lake, river; blast-weights.

1. Introduction
Optical cables are installed in ducts by blowing over > 3 decades. Impressive results have been obtained, 3.8 km in one blow by Trafikverket (Sweden 2015) [1] and even 5.3 km in one blow by EE Energia Engiadina (Switzerland 2019) [2]. With floating (water instead of air) even a longer distance has been reached, 12.4 km in one float by Nexans (Zurich, Switzerland 2019) [3]. Of course not too many bends shall be present in the trajectory and they may not be sharp. Also sections marked as straight might be critical: how straight is straight? Will undulations or micro-undulations be present? The latter may occur when bundles of microducts (outside diameter <16 mm), are direct-buried into the ground, in trenches, then filling up and compacting the soil, at the same time the cables filling up the microduct space for more than 75% in diameter [4]. Micro-undulations are not the subject of this paper, only natural (macro) undulations. Ducts will show such undulations when paid out from a reel. When the quality of the ducts is good and laying is done correctly (e.g. not pulling over the duct from a flat non-rotating reel) such undulations do not cause cable blowing or floating problems after plowing the ducts into the ground, laying them in trenches, gutters, in larger ducts or in tunnels. In this paper the effect on blowing and floating distance is studied of duct undulations caused by periodically fixing the duct by clamps, e.g. to tunnel walls, under bridges, or held in place at sea, lake or river bottoms by blast-weights. It is known that such periodical fixing sometimes causes problems, but no guidelines with underlying theory is known yet, at least not how it influences blowing distances. In Figures 1 and 2 hanging ducts in tunnels are shown at CERN (blowing record of that time of 3.6 km [5]) and in Zurich (12.4 km floating record [3]), respectively, both with the duct hanging with span of 1.5 m.

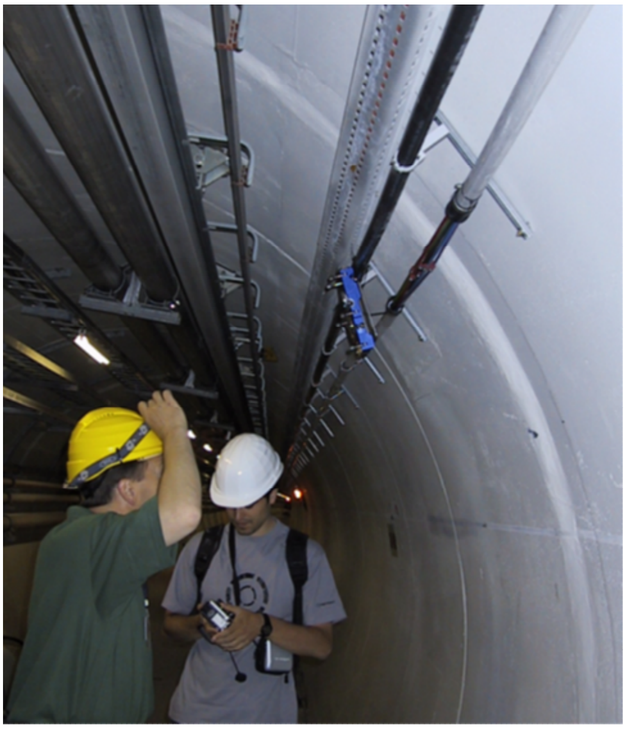

Figure 1. Duct hanging in tunnel of CERN, span 1.5 m. Here the (microduct) cable blowing record of that time of 3.6 km was reached.

Figure 2. Duct hanging in tunnel in Zurich (Switzerland), span 1.5 m. Here the cable floating record of 12.4 km was reached. The cable came out with 50 m/min.

A duct will sag between its clamping points by its weight. This sag is counter balanced by the stiffness of the duct and the axial (pre-) tensile force in the duct. A numerical calculation is done taking into account both counter effects. Temperature variations are also of influence. The result is a combination of amplitude and period of the duct undulations as a function of different parameters. Their effect will be evaluated on the blowing and floating distances, calculated with the software JetPlanner [6] (a summary of the theory will be given too). The effect of sag of the cable itself is in principle also of influence, just as creep of the duct. However, these effects are not taken into account in the calculations but evaluated qualitatively for different situations.

2. Analysis

It is analyzed what the effect is on the cable blowing and floating length when a duct is fixed with regular span, taking into account pre-tensioning and temperature effects.

2.1 Hanging of Duct

When a duct with stiffness B_d is hanging under its own weight (W_d per unit of length) between supports with intermediate distance d, the sag S_{dB} (determined by the stiffness B_d of the duct) in the middle, in case no pulling forces are applied to the duct, is given by (A8) of Appendix A:

$$S_{dB} = \frac{W_d}{B_d}\frac{d^4}{384} \qquad (1)$$

A duct with internal diameter D_d and external diameter OD will have the following weight W_d and stiffness B_d:

$$W_d = \frac{\pi}{4}\left(OD^2 - D_d^2\right)\rho g \qquad B_d = -\frac{\pi}{64}\left(OD^4 - D_d^4\right)E \qquad (2)$$

Here ρ is the density (HDPE 0.94 g/cm^3) and E the Young´s modulus (HDPE 0.8 GPa) of the duct. For a 32/26 mm duct a weight W of 2.52 N/m and a stiffness B_d of 23.2 Nm^2 follows.

The above calculation was for the situation that no pulling force is applied on the duct when installed. For large spans between the clamps the sag becomes very large, so it will be required to put some pulling force F on the duct. For the case of zero stiffness the sag S_{dF} (determined by the force) of the duct then follows from (A9) of Appendix A:

$$S_{dF} = \frac{W_d}{F}\frac{d^2}{8} \qquad (3)$$

When there is no clear dominancy of the effect of stiffness or tension, a numerical solution of (A5) is required, as is indicated in Appendix A. In Figure 3 the sag S_d is shown as a function of intermediate clamp distance d. For small d the stiffness is dominant, for large d tension takes over and limits further sag. The sag, including both stiffness and tension, will always stay below the blue S_{dB} line (first following for small d) as well as below the red S_{dF} line (eventually parallel to it for large d), see example of Figure 3. Note that thermal effects (expansion and shrinking, see further) will also induce tension effects.

In Figure 4 the sag S_d is shown for different axial forces as a function of intermediate clamp distance d. Note that the span for a negative force F never fully reaches the buckling length b for that force (except for cable weight zero) as given by [4]:

$$b = 2\pi\sqrt{-\frac{B_d}{F}} \qquad (4)$$

This buckling length is 6.77 m and 9.57 m for a (pushing) force F of -20 N and -10 N, respectively. Due to the weight of the duct the buckling asymptote is reached at shorter distances, at 5.45 m and 6.52 m, respectively (at much higher S_d than visible in Figure 4).

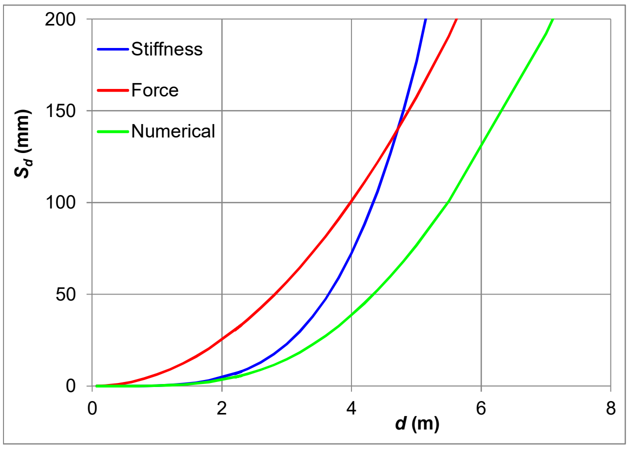

Figure 3. Sag S_d of the 32/26 mm duct as a function of span d for stiffness domination (S_{dB}, no force) and for force (50 N) domination (S_{dF}, zero stiffness). For both effects the numerical solution S_d of (A5) is given.

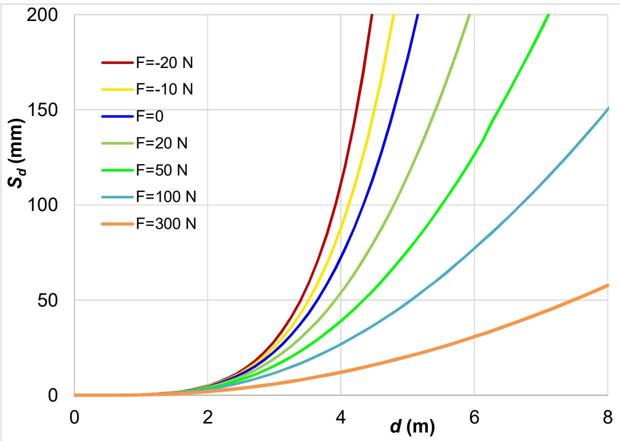

Figure 4. Sag S_d of the 32/26 mm duct as a function of span d for different forces.

2.2 Temperature Effects

When the temperature changes, the length of the duct will also change. This will cause a change in both sag and force. The difference in length of the hanging duct between initial hanging at temperature T_0 and at temperature T_s during service must be known to calculate this.

First the length L_d of the duct in a single duct span is calculated for small y':

$$L_d = \int_0^d \sqrt{1 + y'^2}\,dx \approx \int_0^d \left(1 + \tfrac{1}{2}y'^2\right)dx \qquad (5)$$

For the stiffness dominated situation, the length L_{dB} of the duct can then be calculated using (A7):

$$L_{dB} = \int_0^d \left\{ 1 + \tfrac{1}{2}\left[\frac{W_d}{B_d}\left(-\tfrac{1}{12}d^2x + \tfrac{1}{4}dx^2 - \tfrac{1}{6}x^3 \right) \right]^2 \right\} dx \qquad (6)$$

Carrying out the integration and using (1) to write in terms of sag S_{dB} gives:

$$L_{dB} = \left[1 + \frac{1}{60480}\left(\frac{W_d d^3}{B_d} \right)^2 \right] \cdot d \approx \left[1 + \frac{256}{105}\left(\frac{S_{dB}}{d} \right)^2 \right] \cdot d \qquad (7)$$

For the force dominated situation, the length L_d on one span d follows from the catenary of [7], again for not too large y', and can be written in terms of sag S_{dF} using (3):

$$L_{dF} = \left[1 + \tfrac{1}{6}\left(\frac{W_d d}{2F} \right)^2 \right] \cdot d = \left[1 + \tfrac{8}{3}\left(\frac{S_{dF}}{d} \right)^2 \right] \cdot d \qquad (8)$$

When comparing for the same sags S_{dF} and S_d the lengths L_{dF} and L_d, the latter by carrying out numerical integration, about the same length is found for the catenary. For the numerical example of the 32/26 mm HDPE duct under a tensile force F of 50 N, having the same sags S_{dF} and S_d of 140 mm for span d of 4.72 m, the lengths L_{dF} and L_d of the catenaries are 0.24% and 0.22% longer than the span d, respectively. So, for the same span d and same sag S_d, the total length L_d is about the same for the force dominated and numerical situation. The relation between total length L_d and sag S_d will also be the same for both situations and follows from the general case of (8):

$$L_d = \left[1 + \tfrac{8}{3}\left(\frac{S_d}{d} \right)^2 \right] \cdot d \qquad (9)$$

Compare this to (7) for the stiffness dominated situation, also not differing a lot (8/3 compared to 256/105).

When for initial sag S_{d0} at initial temperature T_0 the total length is L_{d0} and the force F_0, the total length of the catenary will change to L_{d1} and the force to F_1 when the temperature changes to T_1. The relation between L_{d1} and L_{d0} is given by:

$$L_{d1} = L_{d0}\left[1 + k_d\left(F_1 - F_0 \right) + \alpha_d\left(T_1 - T_0 \right) \right] \qquad (10)$$

Here α_d is the thermal expansion coefficient ($1.2 \cdot 10^{-4}$ K^{-1} for HDPE) and k_d the spring constant of the duct, the latter given by:

$$k_d = \frac{4}{\pi\left(OD^2 - D_d^2 \right)E} \qquad (11)$$

For a 32/26 mm HDPE duct the value of k_d will be $4.6 \cdot 10^{-6}$ N^{-1}, i.e. for a force of 50 N the elongation would only be 0.023 %. It also follows that a temperature decrease of 20 °C under constant length would generate a force of 522 N, so temperature effects cannot be ignored.

An example is given of a 32/26 mm HDPE duct, installed at 30 °C (sunny day, black HDPE ducts will then readily get such a temperature or higher) with a span of 4 m under a tensile force of 50 N. According to (A5) the sag of the duct will be 39.1 mm. The length of the duct in one span will then be 4.001 m. When the duct is cooled by 20 °C, pure shrinking would result in a new length of 3.991 m, shorter than the span, not possible of course. The force will also increase, elongating the duct. Iteratively a new balance of force and sag is found: try a force increase and calculate the duct length on one span (shall be higher than the span d) with (10). Next calculate the sag that would follow from this combination of force and span with (A5) and from that the duct length on one span with (9). Vary the force increase until this duct length is the same as the one directly obtained with (10). For this case a force increase of 469 N follows. The total duct length on one span here is 4.00004 m. This would follow directly from (10) for simultaneous cooling 20 °C and increasing the force with 469 N from 50 N to 519 N. For this new force a sag of 7.7 mm follows with (A5) and from that again the same duct length 4.00004 m from (10). So, the sag decreased considerably, from 39.1 mm to 7.7 mm. Clearly cooling has a large effect on the sag!

In Table 1 sags S_d are given for a 32/26 mm duct for different temperature changes ΔT and different initial forces F_0, 3 tables for different spans d. When the equilibrium force F becomes negative (compressive) the sag S_d in Table 1 is indicated in blue.

Table 1 Sags S_d (mm) for different spans d, different initial forces F_0 and different temperature changes ΔT.

***d* = 2 m**

ΔT (K) \ F_0 (N)	10	20	50	100	200	500	1000
-30	1.1	1.0	1.0	1.0	0.9	0.7	0.5
-20	1.4	1.4	1.3	1.3	1.1	0.9	0.6
-10	2.1	2.1	2.0	1.8	1.5	1.1	0.7
0	4.3	4.2	3.7	3.2	2.4	1.5	0.9
10	22.1	20.9	17.3	11.8	6.0	2.2	1.1
20	46.1	45.4	43.1	39.0	29.8	4.9	1.5
30	62.4	61.8	60.1	57.2	50.9	25.5	2.3

***d* = 4 m**

ΔT (K) \ F_0 (N)	10	20	50	100	200	500	1000
-30	5.8	5.6	5.2	4.7	4.2	3.2	2.4
-20	8.9	8.4	7.7	6.6	5.6	4.0	2.8
-10	18.7	16.7	13.5	11.0	8.5	5.4	3.3
0	61.8	53.9	39.1	26.9	16.7	7.7	4.1
10	102	95.8	84.4	71.9	51.2	14.3	5.4
20	131	127	118	109	93.8	42.0	8.0
30	156	152	145	124	111	87.1	15.2

***d* = 6 m**

ΔT (K) \ F_0 (N)	10	20	50	100	200	600	1000
-30	153	52.8	16.3	12.1	10.1	7.1	5.5
-20	197	114	28.0	17.5	13.6	8.6	6.4
-10	234	166	67.0	31.6	21.0	11.1	7.7
0	266	208	127	77.2	43.8	15.6	9.6
10	294	243	176	137	100	26.1	12.8
20	321	274	216	184	155	60.7	19.2
30	345	302	250	222	199	122	36.8

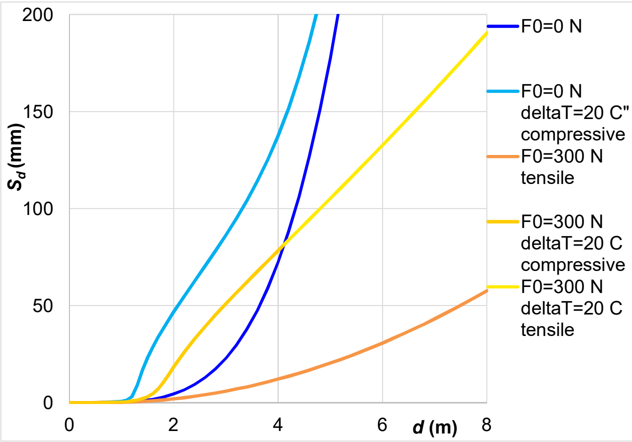

Figure 5. Example of Sag S_d as function of span d for tensile force 0 and 300 N on 30/26 mm duct during hanging and effect of temperature increase ΔT of 20 °C

In Figure 5 an example is shown for the 32/26 mm duct which is initially hung with a tensile force of 0 N and 300 N, respectively. The sag S_d is shown as a function of the span d for these (initial) axial forces F and this is also done after a temperature increase of 20 °C for these cases (where F changes). Note how the yellow line (initial force 300 N, increased in temperature by 20 °C) changes from compressive to tensile force when crossing the blue line (initial force 0 N).

It can be seen that the sag rapidly increases from a clamping length around 1 m. A sec force of 522 N to balance the temperature increase of 20 °C follows from (10) resulting in a buckling length of 1.32 m, as follows from (4). When buckling starts length is gained easily, no problem to create the 0.24% excess length needed according to (10), which is reached at a sag of 40 mm according to (9). The force drops, now below the buckling threshold. A new equilibrium is established, with a force of 484 N. The "missing" 38 N would according to (10) need 0.018% excess length for which according to (9) a sag of 11 mm would be sufficient. But, this is already more than the free space of the cable in the duct. So, blowing length starts decreasing immediately when the span passes the buckling length belonging to the force created by the temperature increase.

Note that the sag may become high when forces are compressive. So, installing at low temperatures brings the risk that upon heating the sag becomes so high that blowing distances are reduced considerably. The same risk occurs when due to creep in the cold season the ducts became longer, leading to blowing problems in the hot season. So, temperature variations might cause considerable sag also when the duct was installed in favorable conditions.

2.3 Effect of Water

When the cable is floated in with water the duct becomes heavier, increasing the sag. It might also affect the temperature of the duct, indirectly influencing the sag. The weight of the duct increases from 2.52 N/m (empty) to 7.73 N/m (with water). It is considered that the duct is empty before installation of the cable and the first water filling occurring during floating in the cable. Two situations are considered, same temperature during cable floating as during installation of the duct and a reduction in temperature of 20 °C during floating. The latter is more interesting to study than temperature increase, because this is a feature which can be used advantageously with floating. And temperature increase does not do a lot here, as the axial forces in the duct remain tensile, because of the extra stretching of the duct due to water loading.

In Figure 6 the effect on the sag S_d of the duct from filling and cooling the duct with water is shown. The initial (empty) hanging conditions (same as in Figure 5) for 0 N and 300 N pre-tension are also shown as a reference. Clearly the sag increases after filling the duct with water. This effect becomes less when use is made of the cooling ability of the water flow. Better also install the duct initially with pre-tension. It is interesting to see what happens when there is no pre-tension and the duct is later cooled by 20 °C with the water flow. From (10) it follows that this is equivalent to a tensile force of 522 N. This is the force seen at very small span. Increasing the span will let this force grow a little bit, because of the extra force from the weight of the duct span. The max force (0.5 N higher) is reached at a span of 1.6 m. After that the force goes down again, only 29 N left at a span of 8 m. This can be explained by the fact that there is enough "room" in the long (sagged) duct span to accommodate this small expanded length of only 0.24% for 20 °C temperature drop.

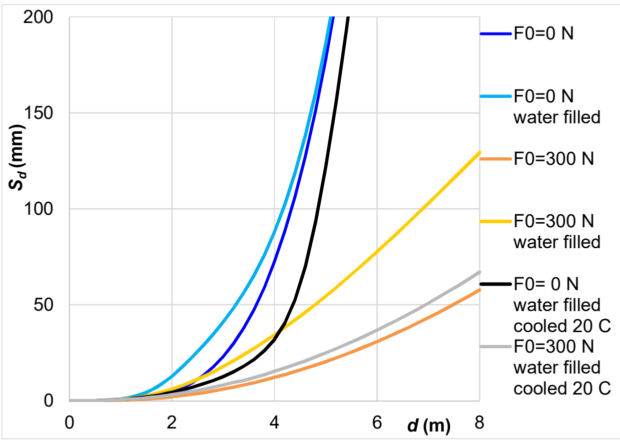

Figure 6. Example of Sag S_d as a function of span d for tensile force 0 N and 300 N on a 30/26 mm duct during hanging and how this is influenced by filling and cooling by ΔT of 20 °C with water

2.4 Effect on Blowing and Floating Distances

With blowing and floating the cable is installed with help of fluid drag forces, generated by injecting fluid under pressure in the duct. When the fluid is a gas (air) the technique is called blowing, when it is a liquid (water) it is called floating. To generate drag forces the fluid has to flow (much) faster than the cable, so no pig is used at the front end of the cable. Because friction caused by gravity is locally compensated by the fluid drag forces hardly any tensile force is built-up in the cable and, hence, the capstan effect is suppressed [6]. For this reason surprisingly long installation lengths can be reached while tensile forces on the cable remain low. With floating the distances reached are even longer because of buoyancy reducing the friction caused by gravity. Additional advantages (of fluid drag methods over pulling the cable) are that the installation is single step (no winch rope to be installed first) and done from a single location (no winch and personnel needed at other end of duct), making it the standard technique worldwide.

Blowing and floating distances can be calculated using the theory of [6]. Here the (pushing) force build-up dF_c/dx of a cable installed in a duct follows from:

$$\frac{dF_c}{dx} = fF_n + W_c \sin\alpha - \frac{dF_{drag}}{dx} \quad (12)$$

Here f is the coefficient of friction, F_n the normal force between cable and duct, $W_c$sinα the force to bring the cable with weight W_c per unit of length uphill (slope α) and dF_{drag}/dx the drag force on the cable exerted by the fluid flow. Note that for floating the effective weight of the cable must be corrected for buoyancy. The normal force F_n is given by:

$$F_n = \sqrt{\left(W_c \cos\alpha\right)^2 + \left(\mathrm{T}F_c - W_B\right)^2 + \left(\mathrm{B}F_c^2\right)^2} \quad (13)$$

Here F_c is the axial (pushing) force in the cable, T the change in direction of the cable in the duct per unit of length (only vertical undulations caused by the sag S_d of the clamped duct considered here), W_B the effect of the cable stiffness B_c in these undulations and B a constant for the effect of buckling during pushing of the cable (which is zero in case of pulling):

$$\mathrm{T} = \frac{8\pi A_{eff}}{d^2} \qquad W_B = \frac{3A_{eff}B_c}{2\left(d/4\right)^4} \qquad \mathrm{B} = \frac{D_d - D_c}{\pi^2 B_c} \quad (14)$$

Here D_c is the cable diameter. The effective (vertical) cable amplitude A_{eff} in the clamped duct with sag S_d is given by:

$$A_{eff} = \tfrac{1}{2}\left(S_d + D_d - D_c\right) \text{ when } \mathrm{T}F_c > W_B \quad (15)$$

$$A_{eff} = \tfrac{1}{2}\left(S_d - D_d + D_c\right) \text{ when } \mathrm{T}F_c < W_B \text{ and } S_d > D_d - D_c \quad (16)$$

$$A_{eff} = 0 \quad \text{ when } \mathrm{T}F_c < W_B \text{ and } S_d \le D_d - D_c \quad (17)$$

The drag force dF_{drag}/dx on the cable exerted by the fluid flow is given by:

$$\frac{dF_{drag}}{dx} = \tfrac{1}{4}\pi D_c D_d \frac{p_i^2 - p_a^2}{2l\sqrt{p_i^2 - \left(p_i^2 - p_a^2\right)\frac{x}{L}}} \quad \text{(blowing)} \quad (18)$$

$$\frac{dF_{drag}}{dx} = \tfrac{1}{4}\pi D_c D_d \frac{p_i - p_a}{L} \quad \text{(floating)} \quad (19)$$

Here p_i is the fluid pressure at the entrance of the duct, p_a the fluid pressure at the end, x the position in the duct and L the length of the duct (which is open at its end).

Note that the cable stiffness friction W_B grows very fast when the span d decreases (inversely proportional to d^4). The sag S_d of the duct first grows proportional to d^4 (duct stiffness dominated) but for larger d it grows with d^2 (duct pulling force dominated), so there might be a window where a shorter span d gives more stiffness friction W_B. On the other hand there is also a window with small d and S_d where the cable can find a straight path when not pushed too hard. Solving the differential equation (12) is the only way to find out which span d is best and how much effect there is on the blowing or floating distance L.

2.5 Example with Blowing

For a cable with diameter D_c of 20 mm the undulations in the 32/26 mm duct are just not seen (sag of 6 mm and small pushing force F_c on cable) for a force zero on the duct when the span is 2.15 m. For a duct pulling force of 300 N this is the case for a larger span of 3 m. However, when the temperature increases by e.g. 20 °C the sag increases to 49 mm (duct pushing force 222 N) and 51 mm (duct pushing force 56 N) for the 2.15 m and 3 m span, respectively. So, for cases where the hanging duct is exposed to outside environment the advantage of not seeing the undulations for the short span totally disappears! In this case the short span may only create extra cable stiffness friction.

In Figure 7 the reduction of the blowing length is shown for the example of Figure 5 for the 20 mm cable, with weight W_c of 3.2 N/m and stiffness B_c of 10 Nm^2 and coefficient of friction f of 0.08, blown with 12 bar and 500 N pushing force. First the effect on the blowing length is shown for the initial hanging situation (before the temperature rise). When no tensile force is applied on the duct the blowing length rapidly decreases after the sag becomes larger than the free space of the cable in the duct at a span d of 2.15 m. When a tensile force of 300 N is applied on the duct this reduction has almost totally vanished. However, creep will cause a reduction of this tensile force in time. It has not been analyzed in this paper how such creep causes reduction in blowing length when the cable is blown after some time after hanging the duct.

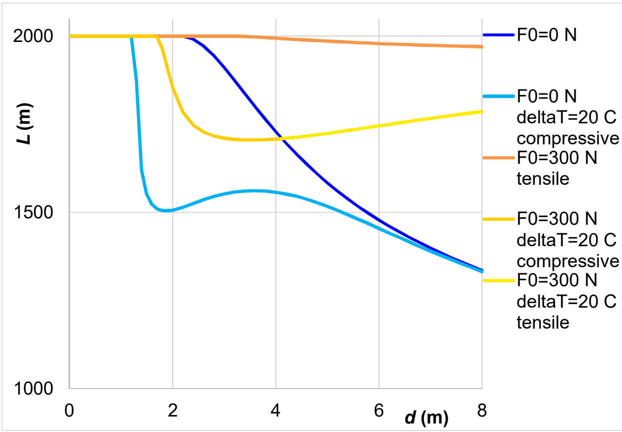

Figure 7. Example of blowing length *L* as a function of span *d* for a 20 mm cable in the 32/26 mm duct for the cases of Figure 5

In Figure 7 the reduction of the blowing length is also shown for the example of Figure 5 after a temperature increase of 20 °C. It is not surprising that the rapid increase in the sag of the duct causes a very rapid decrease of the blowing length (accelerated by the fact that the first part of the rapid increase in sag the blowing length is still unchanged because of the free space of the cable in the duct). It is interesting to see that around a span d of 2 m (a bit earlier for initial force of 0 N during hanging) a local minimum is reached in the blowing length, recovering a bit for larger d and then not changing a lot anymore (still recovering for initial force of 300 N, slowly decreasing for initial force of 0 N, the latter approaching the value without temperature increase when thermal extension of the duct becomes small compared to excess length from the initial sag).

2.6 Sag of the Cable

Until now the sag of the cable itself was not considered. However, for comparable tensile force the sag in the cable is usually larger than that of the duct! For the example cable and duct the sag of the cable is about 3 times that of the duct for a force of 0 N for both. This means that the cable will not experience the stiffness friction as calculated in the previous section, but just follows the bottom of the undulating duct. But, the axial force in the cable is in no way coupled to the axial force in the duct. During blowing the axial force in the cable varies from compressive close to the cable feed side until tensile at the cable's front length [6]. An important region for the performance of cable blowing is the long extending low axial force region. And in this region the cable sag is small compared to the duct sag in case the latter experienced a temperature increase after initial installation of the duct.

One might question whether the cable does experience excess friction when simply following the bottom of the undulating duct without experiencing normal forces from the duct to bend it. Indeed there is no bending stiffness friction like W_B in (14). But, dissipation of the continuously bent cable causes effective friction, as is known from pulling cables over rollers. Here effective friction is known to increase with roller distance (so is not caused by friction in the rollers itself). The friction of the continuously bent cable in the duct will be less when most of the cable sag is supported by the duct with smaller sag. Blowing tests are proposed to find all details.

When the cable is floated in with water instead of blown in with air, the effective weight of the cable becomes less. Besides that the sag of the duct increases because it becomes heavier, also the sag of the cable decreases. Now the calculations without cable sag become relatively more accurate.

2.7 Example with Floating

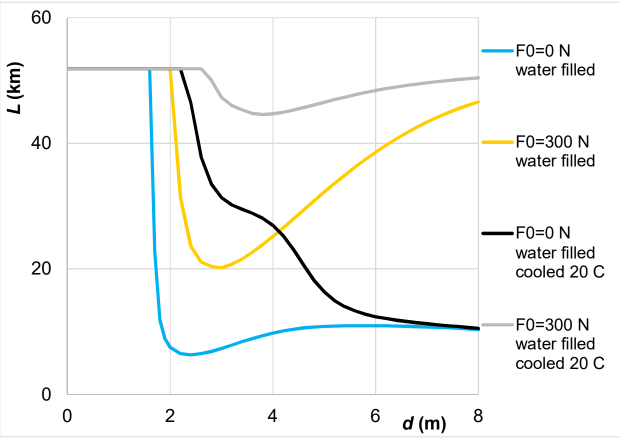

Figure 8. Example of floating length *L* as a function of span *d* for a 20 mm cable in the 32/26 mm duct for the cases (floating, so only water filled) of Figure 6

Buoyancy makes that floating with water reduces the effective weight of the cable, having an advantageous effect on the distance that can be reached. It shall be noted that for long lengths the water speed is low: for the example with 20 mm cable in 32/26 mm duct a water pressure of 1 bar per km gives a water speed of 16 m/min (for a 40/33 mm and 50/40 mm duct that would be 24 and 29 m/min, respectively). Equation (19) is only valid for a water speed >> cable speed. Calculation is done with this condition, bearing in mind that extra water pressure is needed on top of the 12 bar in the example to give the extra water speed difference with the cable. As this requires extra force to push the cable inside the pressure zone calculation is done with a bit lower remaining pushing force of 377 N (exactly "back pressure force" for the 12 bar in the example), so $F_c = 0$. Equations (12), (13) and (19) can then (for $dF_c/dx = 0$, i.e. F_c remains zero) be simplified to:

$$L = \frac{\pi D_c D_d \Delta p}{4 f \sqrt{W^2 + W_B^2}} \tag{20}$$

The results are shown in Figure 8. For small span d, where the sag S_d is still smaller than the free space of the cable in the duct, the calculated floating length is 51.8 km for the treated example! This supports the large distance of already 12.4 km found in practice for floating in a trajectory without many (sharp) bends [3]. Here a water pressure of 20 bar was used and the cable came out with 50 m/min.

As soon as the sag S_d becomes larger than the free space of the cable in the duct, the floating length decreases rapidly. When there was no pre-tension on the duct during installation, the floating length drops to about 6.26 km for the worst case span of 2.4 m. When the duct is cooled there is no such local minimum, only a hump, and the minimum is 10.5 km for a span of 8 m. When a pre-tension of 300 N is used during installation of the ducts the floating lengths are always longer than 20 km (and this is again in a local minimum).

Creep can spoil the beneficial effect of pre-tensioning, not further treated in this paper. But, in all cases extremely good floating properties are to be expected when selecting a span d below the critical value of 1.6 m for this example (and proportionally larger for larger ducts). Note that in such favorable conditions excess pushing forces reach far. That explains that in some favorable floating installations the cable speed remains high and the cable comes soon after the first water comes out. That is also what occurred in [3].

3. Conclusions

From the above it might become clear that exact calculation of the reduction in blowing and floating length for a duct fixed at regular intervals is even more complex than treated so far. However, some rules of thumb follow from the analysis:

- When the duct is in an environment of constant temperature the blowing distance is hardly affected for span < 2 m.
- When the duct is in an environment of constant temperature and hung under sufficient axial tensional force the blowing distance is also hardly affected when larger spans are used.
- When the duct is in an environment where the temperature varies (e.g. under bridges) the blowing distance might drop significantly (by ~1/3 in the treated example) already at small spans around 1 m (around the buckling length of the duct when under a pushing force equivalent to the force generated by thermal expansion). For larger spans the blowing distance does not anymore further drop fast, might even recover a bit.
- The above phenomenon might occur not only when the temperature during cable installation is higher than that during installation of the duct, the combination of creep and temperature variation might also cause the same for favorable installation temperatures.
- For large spans the effect of temperature becomes less, especially for low initial axial force in the duct.
- When the cable is floated (with water) instead of blown (with air) all parameters change (the duct becomes heavier, the cable effectively lighter, but the water flow can also influence the temperature, when cooling reducing the sag of the duct).

- Floating lengths can be extremely high (51.8 km in example). In the worst case (span 2.4 m, no pre-tension, no cooling) still even 6.26 km is reached. Selection of a suitable span gives much better results, even better when pre-tensioning the duct. Bad situations can often be saved by cooling with the water.
- The above rules are valid for the example of the 32/26 mm duct. When the duct becomes larger (the cable scaling with the duct) the mentioned span scales proportionally and the forces quadratic with the duct diameter.

4. Acknowledgments

Special thanks to Vitor Goncalves (Plumettaz, Switzerland) for triggering the work of this paper and to Jean Fehlbaum (Nexans, Switzerland) for his involvement in the 12.4 km floating project [3] where duct clamping was designed with help of theory of this paper.

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Appendix A: Numerical Analysis Duct Fixed at Regular Intervals

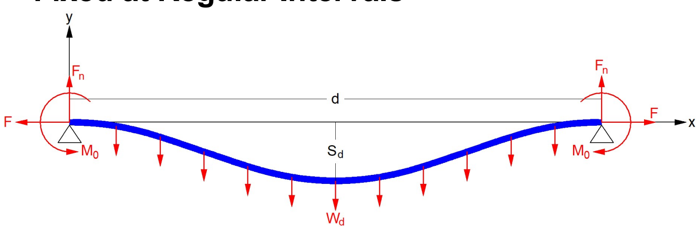

Figure A1. Model for analyzing duct hanging on clamps. For details see text.

In Figure A1 a single duct span is shown. Normal forces F_n from the clamps on the duct balance the total of the distributed weight W_d of the duct and cause bending of the duct and bending moments M_0 at the clamps. At the same time the axial pulling force F in the cable counteracts the bending, also influencing the elastic line of the cable. To analyze this elastic line it is sufficient to only consider normal forces acting on a single span. The total of the distributed weight W_d in the span is equal to the normal force F_n at the ends. Because neighboring spans also start with a normal force only half of this force belongs to the analyzed span:

$$F_n = -\tfrac{1}{2} W_d d \tag{A1}$$

For calculation of the normal forces, only the left half of the span is taken. In this case it is sufficient to consider the forces and bending moment at x=0:

$$M = M_0 - F_n x + \int_0^x W_d ds + yF \tag{A2}$$

Integration of (A2) and using (A1) gives:

$$M = M_0 + \tfrac{1}{2} W_d dx - \tfrac{1}{2} W_d x^2 + Fy \tag{A3}$$

To obtain the elastic line, the following equation must be solved for the curvature K_d of the duct [8]:

$$K_d = \frac{y''}{\left(y'^2+1\right)^{\frac{3}{2}}} = \frac{M}{B_d} \tag{A4}$$

Here B_d is the stiffness of the duct. Combining (A3) and (A4) the following equation is obtained, solved with boundary conditions $y(0) = 0$, $y'(0) = 0$ and $y'(d) = 0$:

$$\frac{y''}{\left(y'^2+1\right)^{\frac{3}{2}}} = \frac{M_0 + \frac{1}{2} W_d dx - \frac{1}{2} W_d x^2 + Fy}{B_d} \tag{A5}$$

In the case that the pulling force F is zero and for small y' (A5) this becomes:

$$y'' = \frac{M_0 + \frac{1}{2} W_d dx - \frac{1}{2} W_d x^2}{B_d} \tag{A6}$$

Integration with boundary conditions $y'(0) = y'(d/2) = 0$ gives:

$$y' = \frac{W_d}{B_d}\left(-\tfrac{1}{12} d^2 x + \tfrac{1}{4} dx^2 - \tfrac{1}{6} x^3\right) \qquad M_0 = -\tfrac{1}{12} W_d d^2 \tag{A7}$$

Integration again with boundary condition $y(0) = 0$ gives y as a function of x as well as the sag $S_{dB} = y(d/2)$:

$$y = \frac{W_d}{B_d}\left(-\tfrac{1}{24} d^2 x^2 + \tfrac{1}{12} dx^3 - \tfrac{1}{24} x^4\right) \qquad S_{dB} = \frac{W_d}{B_d}\frac{d^4}{384} \tag{A8}$$

Here only stiffness B_d counts. For the case that the stiffness B can be neglected and the pulling force F rules the elastic line, the solution for the catenary follows from [7], again for small y':

$$S_{dF} = \frac{W_d}{F}\frac{d^2}{8} \tag{A9}$$

In case both effects from stiffness B and pulling force F are present (A5) is solved numerically, starting with $x = y = y' = 0$. The value of the (negative) bending moment M_0 is still unknown. It is varied until $y(d) = 0$ or $y'(d/2) = 0$. For the example of the 32/26 mm HDPE duct with weight W_d of 2.52 N/m and stiffness B_d of 23.2 Nm^2 with span d of 4 m a sag S_d of 72.5 mm is found for a force F of 0 N (and M_0 of -3.36 Nm), see Figure A2. This is close to the value (72.4 mm) which follows from (A8). For a force F of 50 N a sag S_d is found of 39.1 mm (and M_0 of -2.28 Nm), see also Figure A2. This is less than the value S_{dF} of 100.8 mm for zero stiffness which would follow from (A9). In Figure A2 also the lines are drawn for other values of the force F.

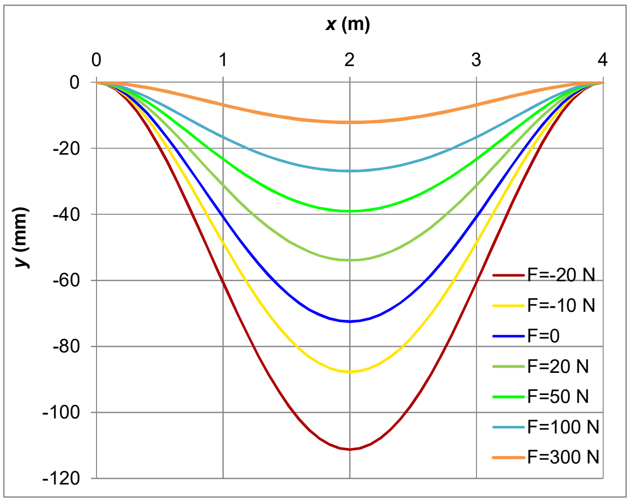

Figure A2 Numerically calculated sag $y(x)$ for duct with weight $W = 2.52$ N/m and stiffness $B_d = 23.2$ Nm^2 for different pulling forces F and span d of 4 m.

6. Author

Willem Griffioen received his M.Sc. degree in Physics and Mathematics at Leiden University (NL) in 1980 and worked there until 1984. Then he was employed at KPN Research, Leidschendam (NL), working in the field of Outside-Plant and Installation Techniques. He received his Ph.D. (Optical Fiber Reliability) in 1995 at Eindhoven Technical University (NL). From 1998 to 2009 he worked at Draka Comteq, Gouda (NL), on connectivity of FttH. Currently he works at Plumettaz SA, Route de la Gribannaz 7, CH-1880 Bex (CH), willem.griffioen@plumettaz.com and is responsible for R&D of cable installation techniques.